Notes on DC Motors

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3 Electric Motors

3.1 Motivation

3.1.1 Historical Context

Rudimentary electric motors have been around since the early 1800's. These motors were not designed to produce useful work; they were only demonstrations of physical phenomenon in electromagnetism. The first practical motors were not developed until the second half of the 19th century, when they began to drive industrial processes such as printing presses, as well as automobiles and trolleys. Most of these early motors were direct current (DC) machines.

The use of electric motors grew as large-scale electric generation and transmission became realized in the 1880's and 1890's. During this time, Nikola Tesla invented the first alternating current (AC) motor. This type of motor, which runs directly from grid power, is still the driving force behind most industrial processes.

3.1.2 Modern Context

Modern motors look a lot like their historical counterparts. The fundamental physical principles governing electric motors have been understood for a long time. The fabrication methods have undergone improvements, but not necessarily radical changes. New permanent magnet materials such as neodymium-iron-boron and advances in steel alloys have definitely contributed to modern electric motor design, making motors more powerful and efficient.

Advances in other fields of technology have expanded the use of electric motors in new and exciting applications. For example, although cars driven by electric motors have been around for over 100 years, modern lithium-ion batteries make electric propulsion a feasible alternative to the internal combustion engine. This, combined with advancements in semiconductors and motor control, has given rise to electric motor-based powertrains like that of the Tesla Roadster[1].

The Roaster is propelled by a 3-phase AC induction motor that is "about the size of a watermelon," according to the Tesla website. The motor can produce up to 288hp and 273ft-lbf of torque. With the help of a modern electronic control system, the motor is capable of producing maximum torque at any speed from 0 to 6,000rpm, and it is able to operate at high power all the way to 14,000rpm. Contrast this to an internal combustion engine, which has a relatively narrow band of peak power and peak torque, and can't produce maximum torque from a standstill. For this reason, the Roadster can outperform other vehicles with a similar power-to-weight ratio even though it only has a single gear.

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[1] http://www.teslamotors.com/

Electric motors can be extremely efficient. The Roadster's motor has a stated efficiency of 88%, compared to 20-30% for a typical internal combustion engine. The electric motor can also operate as a generator, converting some of the vehicle's kinetic energy back into electrical energy to recharge the batteries during braking.

One other practical advantage electric motors have for vehicles is their simplicity: they have only one moving part. This simplicity gives them the ability to achieve high speeds with low wear on bearings and minimal lubrication requirements. Thus, they can be extremely reliable. However, some of the complexity is shifted to the battery and control systems, which need to be made equally reliable for the true advantage to be had.

3.1.3 Functional Definitions

Fundamental Physics

Electric motors convert electrical power into mechanical power through interactions between electrical conductors and a magnetic field. One way of understanding the interaction is through the Lorentz force. The Lorentz force is the force exerted on a current-carrying conductor in a magnetic field, as shown in Figure 1.

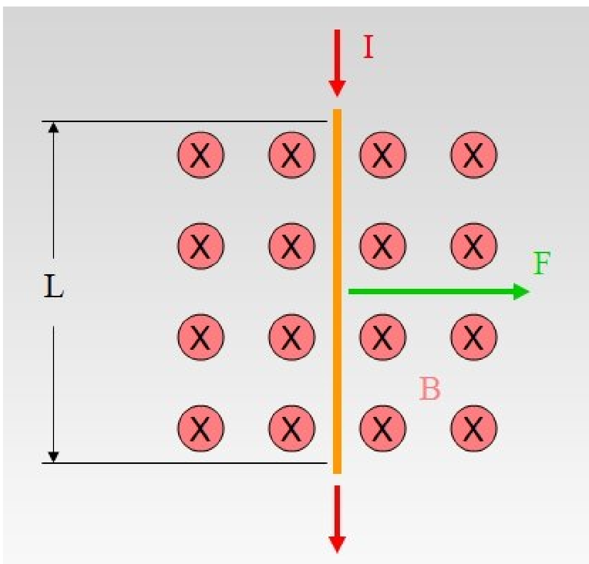

Figure 1: The Lorentz force acts on a current-carrying wire in a magnetic field.

The Lorentz force is defined as:

$$\vec{F} = I\vec{L} \times \vec{B}. \qquad (1)$$

The force is proportional to the current, length of wire, and strength of the field. A single length of wire will not produce much force. For example, consider a wire 10cm long, with 10A flowing through it, in a uniform field of 1T, all typical quantities for a mediumsized robot motor. If the field is assumed to be perpendicular to the wire, the force generated by this wire would be:

$$|F| = (10A)(0.01m)(1T) = 0.1N\,. \tag{2}$$

This force would not be sufficient for most mechanical tasks a motor this size would have to perform. So, motors typically use multiple turns of wire, wrapped around a steel core, to multiply the force. Each turn contributes to the overall force the motor can produce. Together, the turns are called the motor winding. The steel core channels the magnetic flux through the winding, so that the turns do not all have to be directly beneath a magnet to produce force.

The loops of wire comprising the winding interact with the magnetic field through another fundamental principle called Lenz's law. According to Lenz's law, a wire loop moving through a variable magnetic field will experience an induced voltage, called a **back EMF**. Consider the loop of wire in Figure 2:

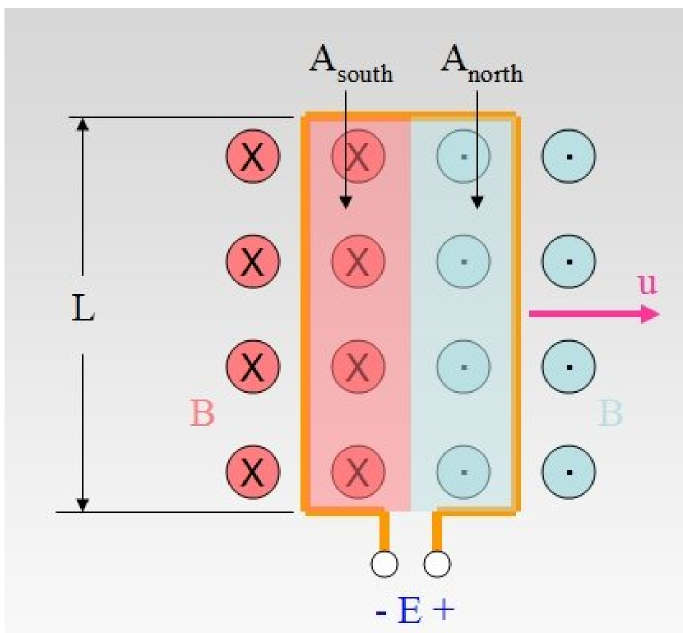

Figure 2: A loop of wire moving through a variable magnetic field experience and induced voltage.

The wire loop is moving with constant velocity *u* through a changing magnetic field, which changes direction from into the page to out of the page, but has the same magnitude, *B*. From Lenz's Law, a voltage *E* is developed across the end points of the loop. This back EMF can be calculated by:

$$E = \frac{d\lambda}{dt} = B\frac{d}{dt}\left(A_{north} - A_{south}\right). \tag{3}$$

If the magnetic field is uniform, the magnetic flux, λ, is the product of the field strength and the enclosed area. (Otherwise, it would be found by a surface integral.) Since the only change in the field is due to the change of signs, and since the wire loop is moving with a constant velocity, the rate of change of enclosed north and south areas can be simplified:

$$E = 2BLu \ . \tag{4}$$

The back EMF produced due to the movement of the wire loop through a magnetic field that changes directions discretely as in Figure 2 is proportional to the field strength, the length of wire loop crossing the field, and the velocity of the wire loop through the field. In the same way, the voltage generated by a rotating motor will be proportional to the field strength, length of the motor, and the velocity of the windings through the field (which itself is a function of the motor's angular velocity and radius).

The voltage generated by a single loop of wire could be quite small. Consider, for example, a loop 10cm long in a 1T field moving a 1m/s. The back EMF would be:

$$E = 2(1T)(0.01m)\left(1\frac{m}{s}\right) = 0.02V \ . \tag{5}$$

However, since a motor normally consists of a winding with multiple turns (loops) of wire in series, the total back EMF can be much larger.

Though the Lorentz force and the back EMF from Lenz's law may seem like independent phenomena, they are in fact two sides of the same coin. The link between equations (1) and (4) is power conservation. To illustrate this, consider the loop of wire in motion and with current flowing through it, as in Figure 3.

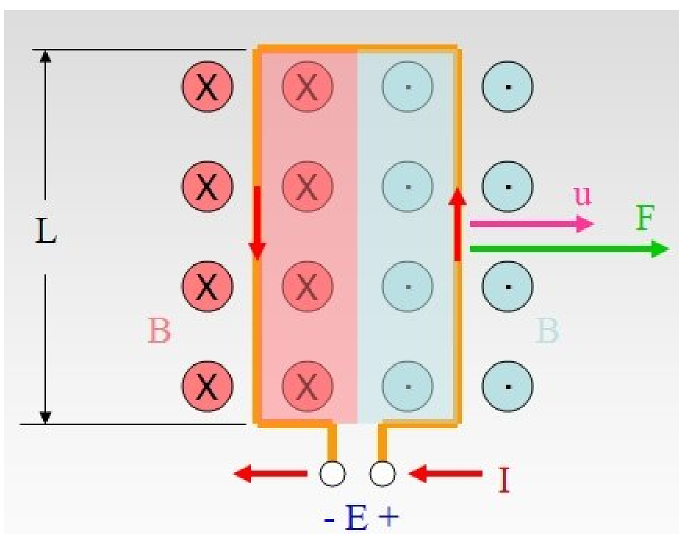

Figure 3: A loop of wire moving through a magnetic field and with current flowing through it.

Both Lenz's law and the Lorentz force equation are at work at the same time. Because there are two lengths of wire contributing to the total Lorentz force, the result from equation (1) is doubled. The direction of the force contribution is the same for both wires, by the right-hand rule. For a coil length of 10cm, a current of 10A, a field strength of 1T, and a velocity of 1m/s, the relevant equations become:

$$L = 10cm\,,\; I = 10A\,,\; B = 1T\,,\; u = 1\frac{m}{s}\,. \tag{6}$$

$$F = 2ILB = 0.2N\,. \tag{7}$$

$$E = 2BLu = 0.02V\,. \tag{8}$$

To see that power is in fact conserved by these relationships, consider the electrical and mechanical power:

$$P_e = IE = (10A)(0.02V) = 0.2W\,. \tag{9}$$

$$P_m = Fu = (0.2N)\left(1\frac{m}{s}\right) = 0.2W\,. \tag{10}$$

Thus, the fundamental physics behind an electric motor of any type is a lossless power transformation from electrical (current and back EMF) to mechanical (force and velocity, or torque and angular velocity). This transformation of power occurs due to interaction between current-carrying wires moving in a magnetic field. Losses associated with the motor are considered to be external to this ideal power transformation. This idea will be further explored in the ideal DC motor model.

Linear motors do exist, but most commonly motors provide rotational actuation, generating a torque at a shaft. Consider the contribution of windings on opposite sides of a simple motor, as in Figure 4. The Lorentz force from each set of windings produces a net torque about the motor's axis of rotation.

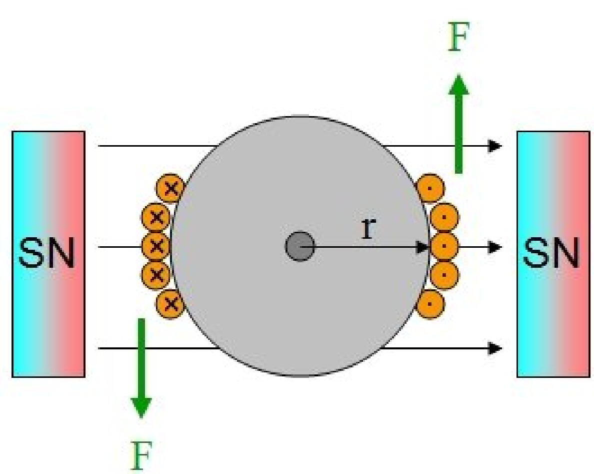

Figure 4: Current-carrying wires on opposite sides of the motor contribute to a net torque by the right-hand rule.

To get from force and velocity to torque and angular velocity, the number of turns of wire and the effective radius, *r*, would be needed. However, this information alone may not be sufficient. Consider that the magnetic field may not be uniform or perpendicular to the wires, that not all wires may be carrying current at once, or that they may be carrying non-uniform amounts of current. Still, the linearity and power conservation of the fundamental relationships suggests that all these unknown factors can be combined into a single constant of proportionality relating torque to current and back EMF to angular velocity. This approach to modeling a simple DC motor will be explored in Section 3.2.2.

Types of Motors

There are a number of overlapping classifications for electric motors. Some are based on physical characteristics of the motor itself. Others consider the nature of the electrical power source driving the motor. Because of the taxonomical overlap, several of the following definitions could be applied to a single motor.

Permanent Magnet Motor: A motor that uses permanent magnets to generate a stationary or rotating magnetic field.

Induction Motor: A motor that uses electromagnetic induction to generate a rotating magnetic field. It has no permanent magnets.

Brushed Motor: A motor that uses mechanical brushes, usually made of graphite, to connect a DC power source to the rotating windings inside the motor.

Brushless Motor: A motor that has no brushes, usually requiring some form of electronic commutation instead.

AC Motor: Generally, a motor driven by a sinusoidal AC voltage source (such as a wall outlet) or a controller which generates such a source. Induction motors and some permanent magnet motors are included in this category.

DC Motor: Generally, a motor driven by a DC voltage source (such as a battery). However, the definition also applies to brushless DC motors, which are typically driven by square-wave AC.

The definitions of AC and DC motors are difficult to separate into two categories because of a significant amount of overlap where the drive method is concerned. Some motors are designed to operate from a DC source but use an electronic controller to produce AC. Table 1 shows a spectrum of motor types from purely AC to purely DC.

Table 1: There is a spectrum of motor types ranging from purely AC driven to purely DC driven.

AC <- -> DC			
AC Induction Motor Can be driven	Permanent Magnet AC Motor Driven by an	Permanent Magnet Brushless DC Motor Driven by an	Permanent Magnet Brushed DC Motor Can be driven

directly from an AC source (wall outlet) or an AC controller.	electronic controller which converts DC to sinusoidal AC.	electronic controller which converts DC to square-wave AC.	directly from a DC source (battery) or a DC controller.

All of the motors used in *2.007*, including motors inside the RC servos, are permanent magnet brushed DC motors. These are perhaps the easiest motors to analyze, and are widely used in small battery-operated applications. The analysis and selection sections of this chapter will focus on permanent magnet brushed DC motors.

3.2 Analysis

3.2.1 Definitions

Back EMF: The voltage produced by the motor as it spins. This is proportional to the speed of the motor.

Brush: A conductive material (usually graphite) used to connect the terminals of the motor to the windings, via the commutator.

Commutator: A segmented conductive section of the rotor to which the brushes make contact. Each segment is connected to portions of the winding.

Eddy current: Circulating currents in conductive materials (copper, steel, aluminum) created by alternating magnetic fields. In a motor, these currents are dissipative and contribute to inefficiency.

Resistance: The electrical resistance of the motor terminals, brushes, and windings. This dissipates power in the form of heat as the motor is running.

Rotor: The portion of a motor that spins.

Stator: The portion of a motor that is fixed with respect to the system on which it is mounted.

Terminals: The external electrical connections leading into the motor windings. Brushed DC motors have two terminals.

Torque constant: A single number, in [Nm/A], relating output torque to input current. It can be found in manufacturer's specifications, or determined experimentally.

Winding: All the turns of wire in a motor. In the case of a brushed DC motor, the winding is on the rotor.

3.2.2 Performance

Simple DC Motor Model

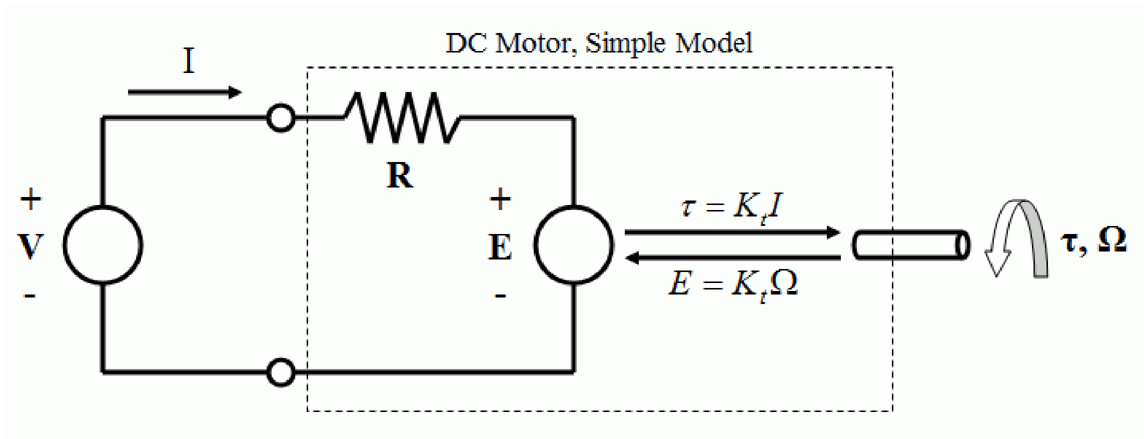

Figure 5: The simple electromechanical model of a DC motor.

Table 2: The definition of symbols used in the simple DC motor model.

Symbol	Description	Units
V	Voltage applied to the motor externally, for example by a battery, controller, or power supply	[V]
I	Current passing through the motor.	[A]
R	Resistance of the motor, which includes the brushes (if applicable) and the windings.	[Ω]
E	Back EMF, the voltage generated by the motor as it spins.	[V]
τ	Torque exerted by the motor on an eternal load through the shaft.	[N·m]
Ω	Angular velocity of the motor shaft.	[rad/s]
K_t	Torque constant, which relates torque to current. By virtue of power conservation, it also relates back EMF to angular velocity.	[N·m/A] or [V/(rad/s)]

Torque is always proportional to current:

$$\tau = K_t I \quad \textbf{(11)}$$

Back EMF is always proportional to angular velocity:

$$E = K_t \Omega \quad \textbf{(12)}$$

.

By virtue of power conservation, these two relationships have the same constant of proportionality. The conversion from electrical power (current and back EMF) to mechanical power (torque and angular velocity) is considered to be lossless. By substituting either relationship into the other, power conservation is clearly shown:

$$\tau = K_t I = \left(\frac{E}{\Omega}\right) I . \tag{13}$$

$$\tau\Omega = EI . \tag{14}$$

Both sides of the resulting equation have units of power: mechanical power at the shaft equals electrical power *at the back EMF*. Note that this is not equal to electrical power at the motor terminals. This is due to power lost in the series resistance between the applied voltage and the back EMF. The power dissipated in this resistance is proportional to the square of current:

$$P_r = I^2 R . \tag{15}$$

Since the resistance generates heat in the motor windings, motor heating is proportional to the square of current; a two-fold increase in motor current will create a four-fold increase in motor heating. Motor efficiency in the simple model is entirely determined by the current. Assuming operation as a motor (not a generator) power input is electrical and power output is mechanical. The efficiency can be represented based on purely electrical variables by substituting back EMF power for mechanical power as in equation (14):

$$\eta = \left(\frac{P_{out}}{P_{in}}\right) = \left(\frac{\tau\Omega}{IV}\right) = \left(\frac{IE}{IV}\right). \tag{16}$$

The back EMF can then be rewritten as a function of applied voltage, current, and resistance:

$$E = V - IR . \tag{17}$$

Substituting (17) into (16) gives the efficiency of the motor:

$$\eta = \left(\frac{IE}{IV}\right) = \left(\frac{I(V-IR)}{IV}\right) = \left(1 - \frac{I^2 R}{IV}\right) = \left(1 - \frac{P_r}{P_{in}}\right). \tag{18}$$

As stated above, all of the losses in this model are accounted for by the power dissipated in the motor resistance, which is proportional to the square of the current. Losses that are not accounted for by this resistance are neglected. Some such losses include bearing friction and eddy current heating, which will be discussed below. More thorough motor models can characterize these other losses, but for many situations the simple model is sufficient.

Ideal Torque-Speed Curves from the Simple Model

Equations (11), (12), and (17) are enough to give a complete description of a DC motor at a single operating point (one torque, speed, voltage, and current). Often, a plot of two or more motor variables over a range of operating points conveys more information. These plots are called motor curves and can be used to quickly solve problems without resorting to the equations.

An important distinction must be made between ideal motor curves, those derived from a model of the motor, and real motor curves, which are often provided by the manufacturer. Real motor curves typically display experimentally measured quantities that reflect true system performance of the motor, accounting for all non-idealities. Ideal motor curves are graphical representations of the motor model, only, and should be used with caution for predicting system performance.

The most common type of motor curve is a torque-speed curve. Though the dependent and independent variables may be reversed in some representations, it is a plot of motor torque as a function of motor speed *at a fixed voltage*. That is:

$$\tau = f(\Omega),\ V = V_0 . \tag{19}$$

With the condition of a fixed voltage, the simple DC motor model can give torque as a linear function of angular velocity:

$$\tau(\Omega) = (K_t I) = \left(K_t \frac{(V-E)}{R} \right) = \left(\frac{K_t V_0}{R} - \frac{K_t^2}{R}\Omega \right) \tag{20}$$

The result of (20) can be understood as two terms: a constant called the **stall torque** and a linear relationship of torque to speed with a negative slope determined by the torque constant and the motor resistance:

$$\tau(\Omega) = \tau_{stall} - K_m \Omega . \tag{21}$$

$$\tau_{stall} = \frac{K_t V_0}{R} = K_t I_{stall} . \tag{22}$$

$$K_m = \frac{K_t^2}{R} . \tag{23}$$

The stall torque, τ_{stall}, has an important physical meaning: It is the torque the motor would produce if the shaft were held stationary (stalled) and the fixed voltage, V_0, were applied across its terminals. At that voltage, this is the largest torque the motor can "hold." If the load torque is greater than the stall torque, the motor shaft will spin backwards (though it will not be operating as a generator in this case, see below). At stall, the motor model reduces to a single resistance and the stall current is just:

$$I_{stall} = \frac{V_0}{R} . \tag{24}$$

Plotting equation (21) gives the motor's torque-speed curve at voltage V_0:

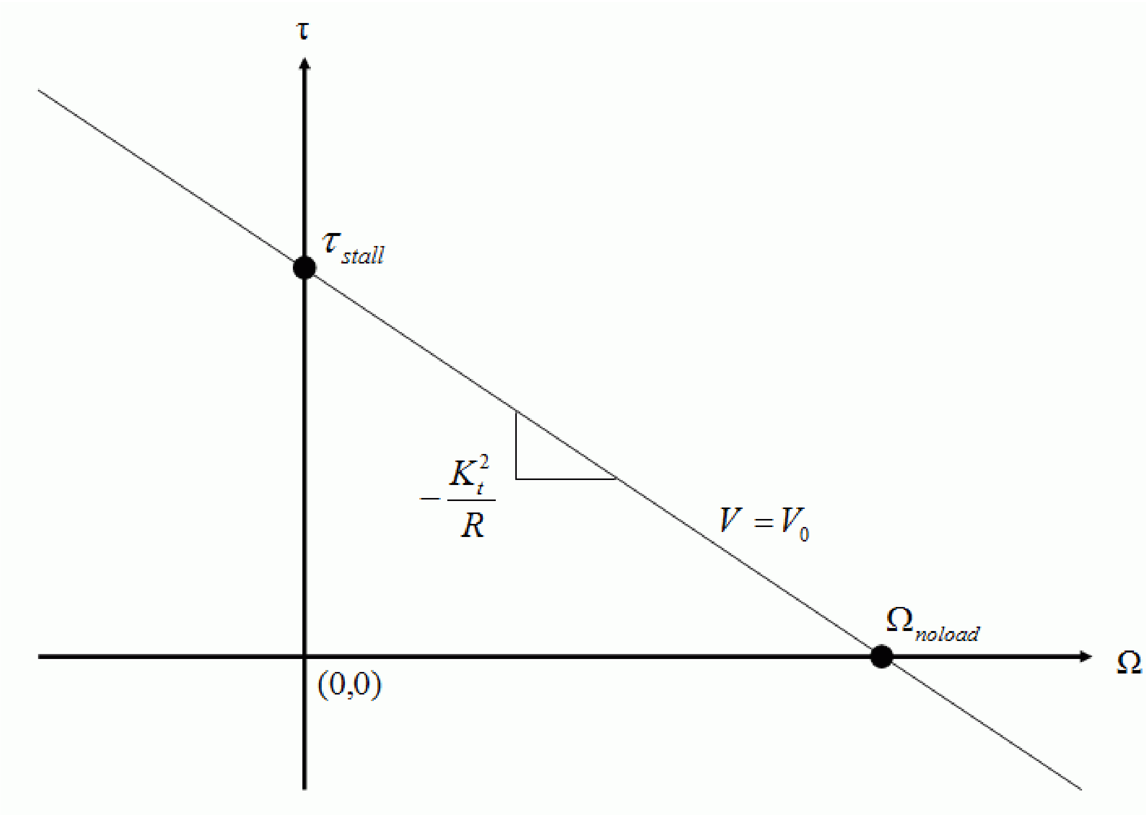

Figure 6: The ideal torque-speed curve of a DC motor at a fixed voltage.

The ideal torque-speed curve contains the same information as Equations (11), (12), and (17), but in graphical form. It represents all possible operating points of the motor given a fixed voltage V_0. One important operating point, $(0,\tau_{stall})$, was already explored and τ_{stall} was explicitly calculated in Equation (24). The other intercept, $(\Omega_{noload},0)$, is called the **no-load speed**, and can be calculated by solving for angular velocity when the torque is zero. From the model, this implies that the current is also zero and that the back EMF is equal to the applied voltage. Thus,

$$E_{noload} = V_0 = K_t \Omega_{noload} . \tag{25}$$

$$\Omega_{noload} = \frac{V_0}{K_t} . \tag{26}$$

Equation (26) shows that a motor with no external load applied to its shaft will increase in speed linearly with applied voltage. Motor controllers that vary the applied voltage are often called "speed controllers" for this reason.

The concept of no-load speed is often confusing because the simple model neglects bearing friction and other loads internal to the motor, such as air gap drag and eddy currents. To factor these loads into the simple motor model, they would have to be considered as additional torque loads, pushing the operating point further up the curve even with no external load connected to the motor shaft.

Typically, internal loads of the motor are small enough that the approximation of zero torque and current at no load works well enough for first-order analysis. However, the contribution of these loads can be experimentally determined by measuring the current draw with nothing connected to the shaft and using equation (11) to solve for the torque due to internal loads. This will reveal the true operating point of the motor, somewhere on the ideal torque speed curve but not at zero torque. This analysis is particularly important for establishing a good model of maximum motor efficiency, which is often determined by these internal loads. A further discussion of efficiency analysis is presented later in the chapter.

Since torque and current are always proportional, the torque-speed curve is proportional to the current-speed curve. Current can be plotted on a secondary y-axis, as in Figure 7.

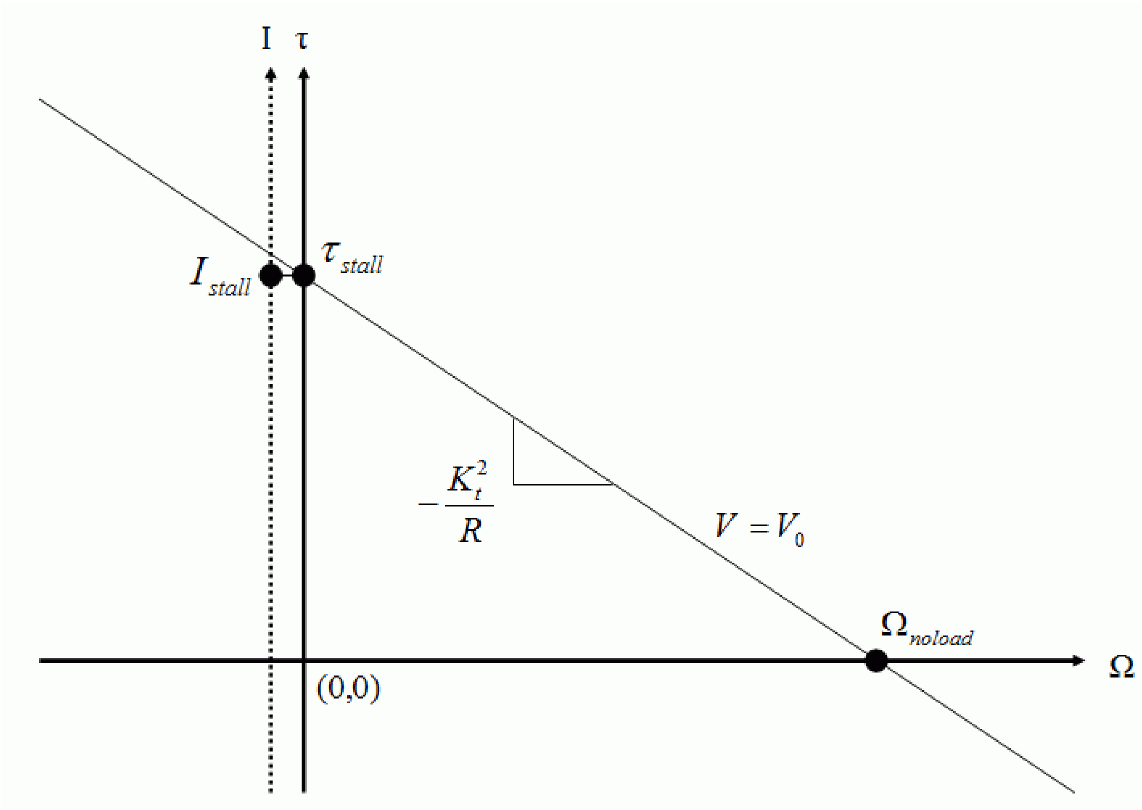

Figure 7: Current can be plotted on a secondary y-axis since it is always proportional to torque.

Regenerative and Plug Braking

Another interesting aspect of the torque-speed curve is that it extends outside of the first quadrant with either negative torque or negative speed operating points. It's useful to consider what these two cases mean physically. The torque-speed curve offers fast insight into this.

First, consider an operating point in the negative torque range, as shown in Figure 8. In this quadrant, torque is negative while speed is positive, thus the motor is outputting negative mechanical power. In other words, it is inputting positive mechanical power; the load is forcing the shaft to spin faster than no-load speed, and in reaction the shaft is trying to slow the load. In the electrical domain, voltage is positive but current is negative. Using the directional convention in Figure 5, current and power are flowing back into the voltage supply. This is **regenerative braking**.

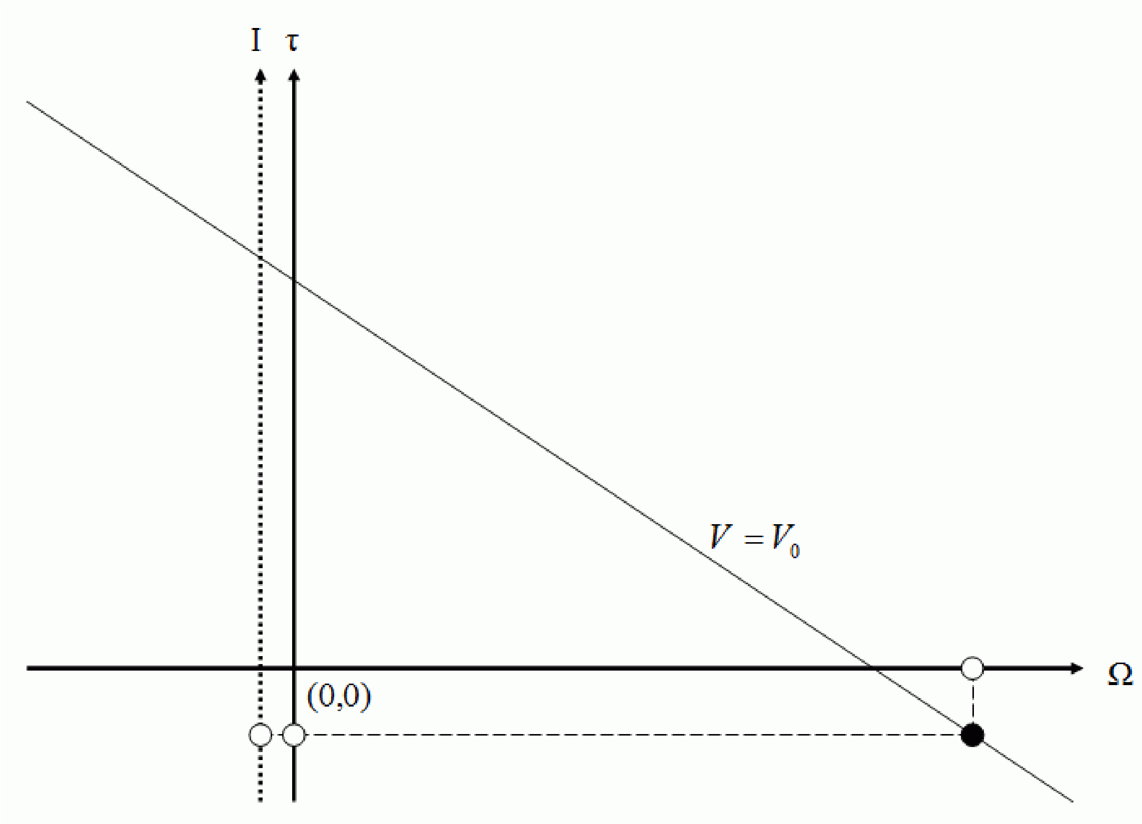

Figure 8: Regenerative braking, where both torque and current are negative.

Next, consider an operating point in the negative speed range, as shown in Figure 9. In this quadrant, speed is negative and torque is positive. (Imagine a car trying to drive up a hill but rolling backwards instead due to insufficient torque.) Thus, the external load is pushing power into the motor shaft. However, in this quadrant the current is still positive, defined as coming out of the voltage source. Power from both the electrical and the mechanical domain are being fed to the motor, which dissipates it all as heat in the resistance. This is called **plug braking**. (It may not seem like braking, since the torque is still positive, but consider the car trying unsuccessfully to climb the hill and imagine instead that it is trying to slow its descent.)

The sign conventions chosen for positive current, speed, torque, and voltage may make the distinction between plug braking and regenerative braking unclear. In these cases, the power flow can always reveal which is the case: Mechanical power output is negative in both braking cases. But, if power is flowing back to the electrical power source, it is regenerative braking. If power is flowing out of the electrical power source, it is plug braking. Figure 10 shows the power flow in these three quadrants of operation.

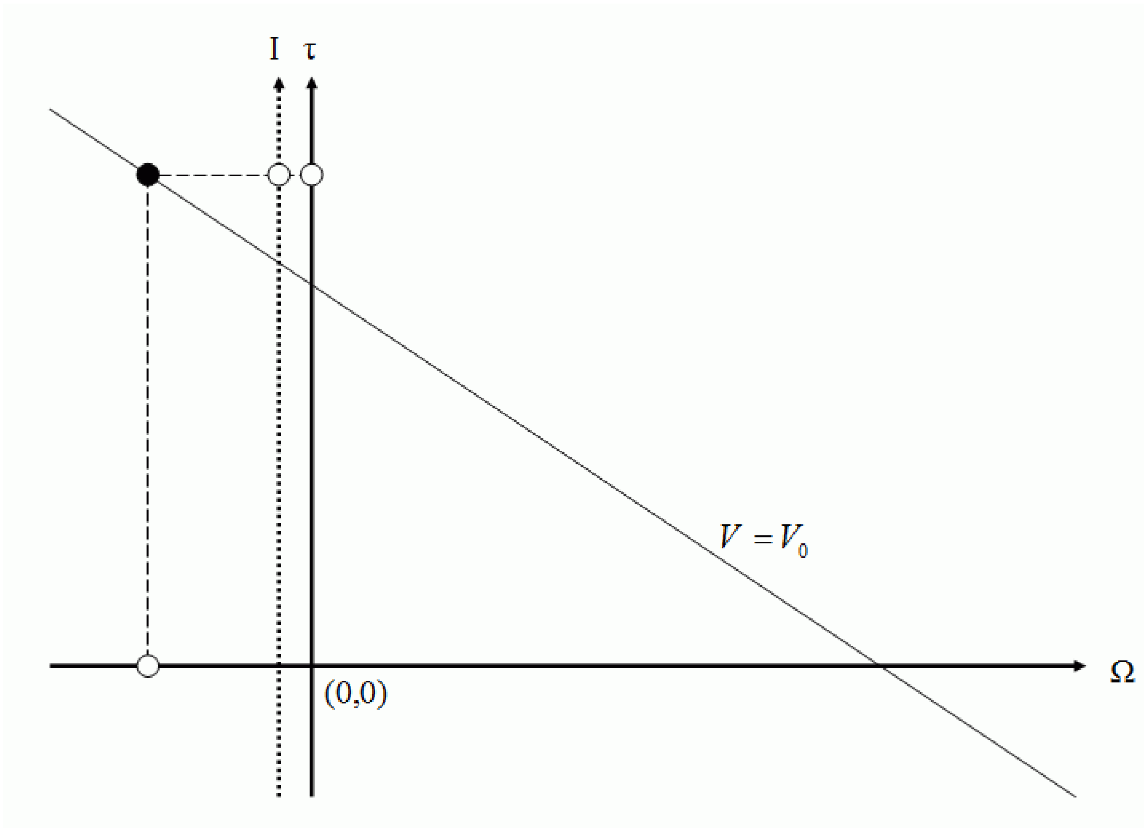

Figure 9: Plug braking, where speed and torque have opposite signs but voltage and current are both positive.

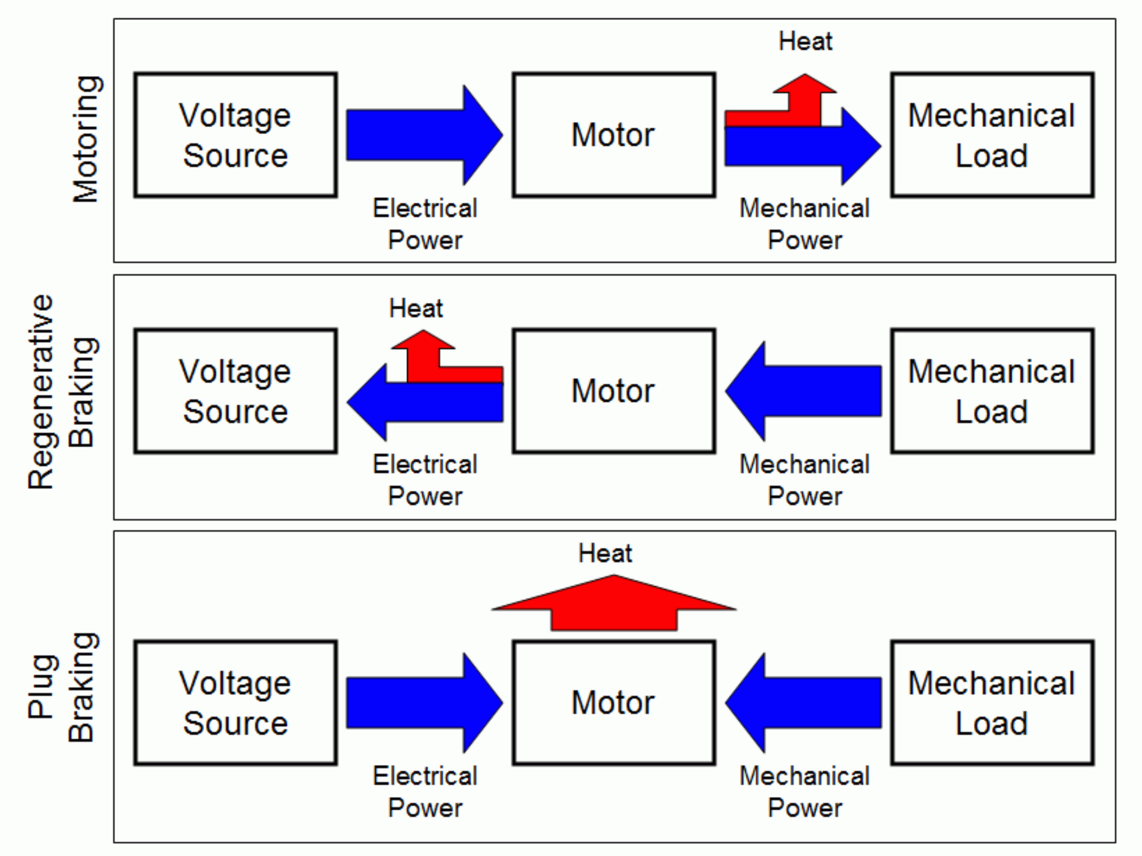

Figure 10: Three possible operating modes of the motor when driven with a positive voltage, shown in terms of power flow.

Power and Efficiency

The torque-speed curve can illustrate power and efficiency of a motor as well. Output power is always equal to the product of torque and speed. It is simple to represent on the torque-speed curve as the area of a rectangle with a lower-left corner at the origin and an upper-right corner on the torque-speed curve at the operating point, as shown in Figure 11.

Maximum power output occurs at half the stall torque and half the no-load speed. This is equivalent to maximizing the area of the shaded rectangle in Figure 11, or maximizing the product of torque and speed given the linear relationship of Equation (21). This is the maximum power output of the motor at the given voltage, and is useful for doing back-of-the-envelope calculations for mechanisms. By substituting electrical variables, the maximum output power can be formulated as a function of voltage and resistance as well:

$$P_{out,\max} = \left(\frac{\tau_{stall}\Omega_{noload}}{4}\right) = \left(\frac{V_0^2}{4R}\right). \tag{27}$$

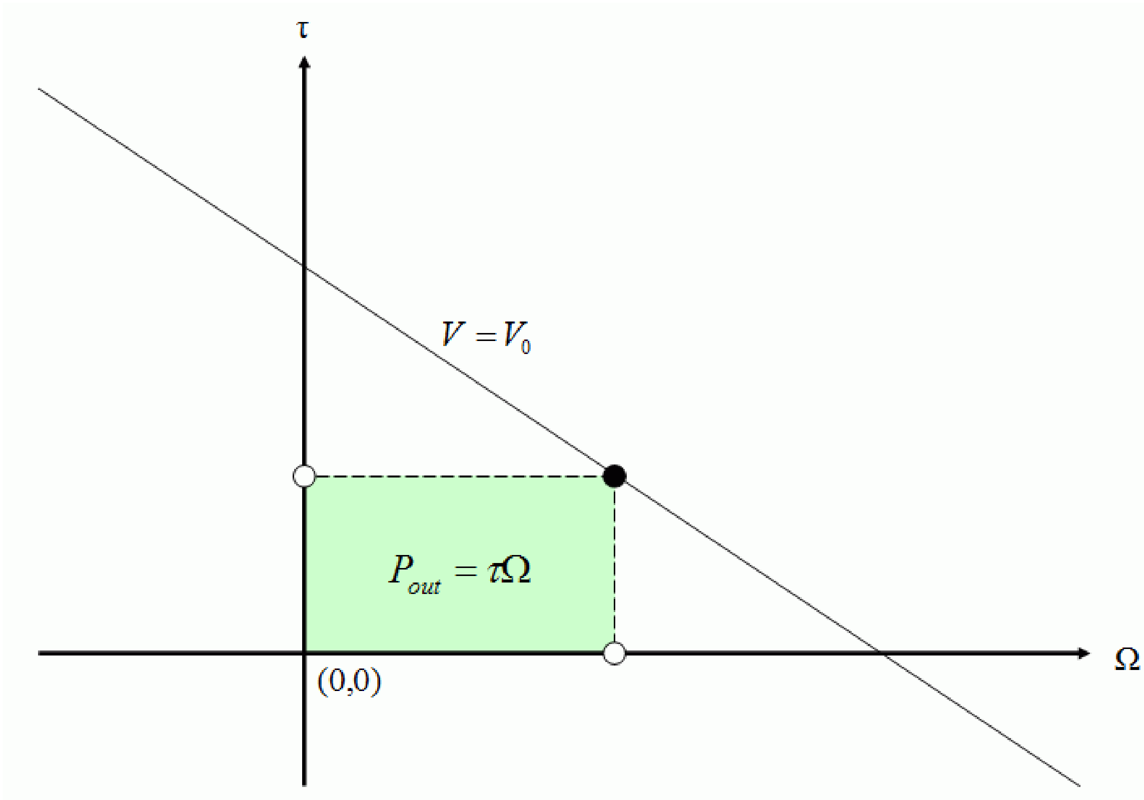

Figure 11: Power is the product of torque and speed. At this operating point, it is the area of the shaded rectangle. Maximum power output occurs at half the stall torque and half the no-load speed.

To visualize power input and efficiency, electrical variables can also be plotted on the torque-speed curve. The proportionality of torque and current was already represented by a secondary y-axis for current. In the same way, the proportionality of angular velocity and back EMF can be represented by a secondary x-axis for back EMF.

Power input at any operating point is the product of current and applied voltage. Applied voltage is equal to the back EMF at the no-load speed, so the power input can be represented on the torque speed curve as a rectangle bounded by the operating current and the x-intercept (no load speed / back EMF) of the torque-speed curve, as shown in Figure 12.

From Figure 12, the relationship between input power, output power, and dissipated power is shown geometrically. Imagine sliding the operating point up or down the torque-speed curve. Toward the high-torque, low-speed side, more electrical input power is dissipated than is converted to mechanical power; the motor runs at very low efficiency. In the middle, the output power is maximized but the efficiency is still a relatively low 50%. As the operating point approaches no-load, the efficiency increases but the power output decreases. Three cases are shown in Figure 13.

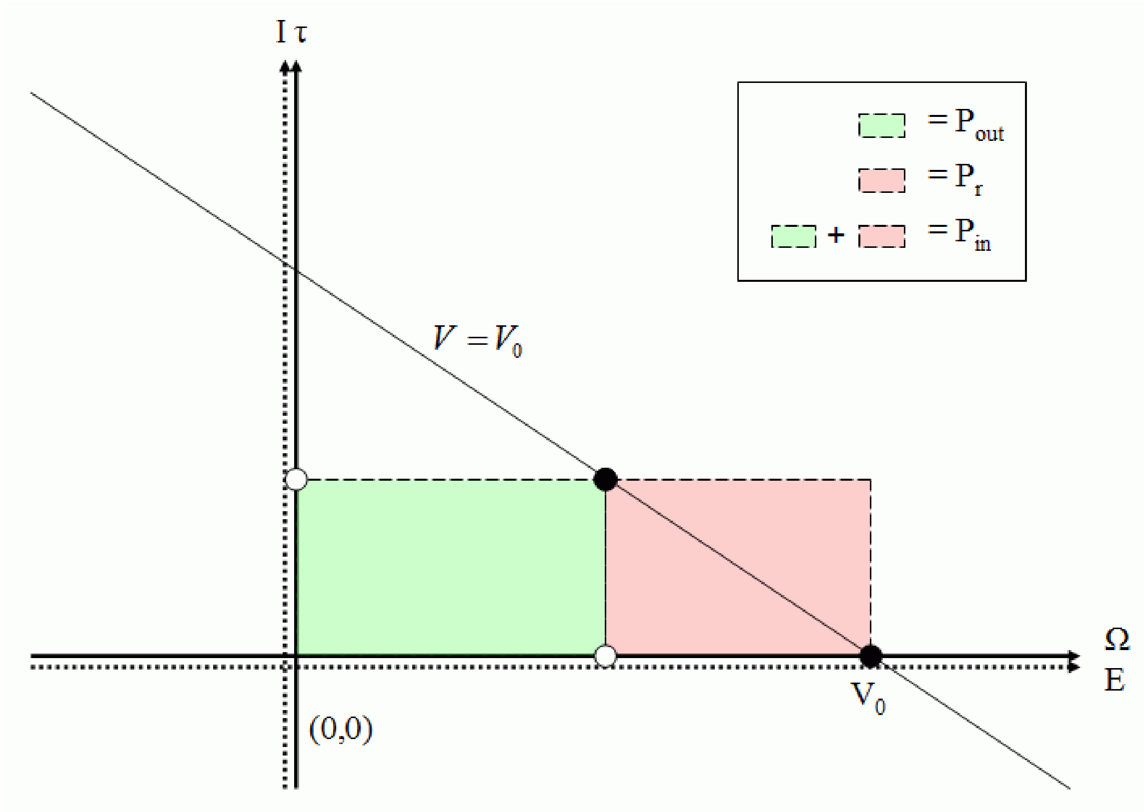

Figure 12: A visualization of the input, output, and dissipated power on the torque-speed curve.

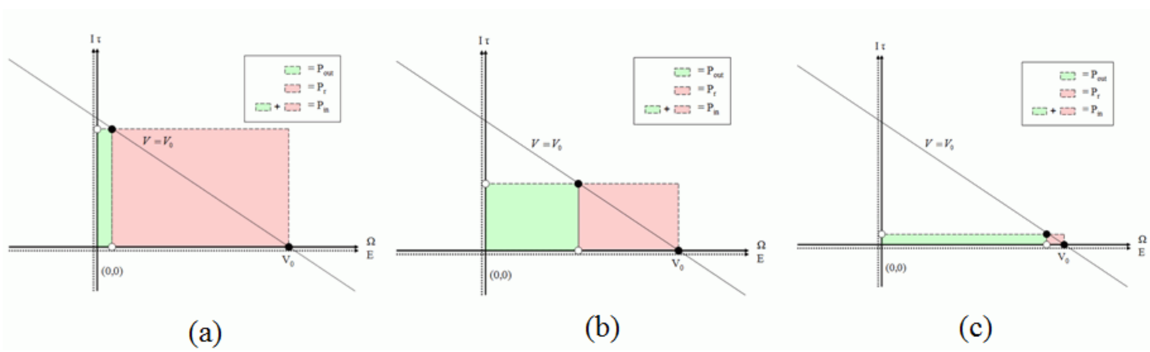

Figure 13: A range of operating points from near-stall (a) to maximum power (b) to near no-load (c) illustrating the power input, output, and dissipation in each case.

Figure 13 illustrates some very practical concepts about motor operation. For one, operation near stall, as in Figure 13a, has little practical advantage: most of the power is being converted to heat by the motor resistance. Running the motor at this operating point is inefficient and will likely result in damage. Often, the solution is to use a larger gear reduction before the final load, which allows the motor to spin faster and with less torque.

Operating at the maximum power point, as in Figure 13b, is usually acceptable for short periods of time. The maximum output power can be useful for accelerating an inertial load or lifting a heavy object. However, the motor is only at most 50% efficient at this point, so it will generate heat and increase in temperature quickly. Running continuously at maximum power will usually result in overheating.

Operating near no-load speed, as in Figure 13c, the motor is dissipating the least amount of power. As a result, it can operate at this point continuously without overheating. Loads that require continuous power, such as pumping, driving a fan, or cruising at a constant speed, will usually necessitate a design in which the motor operates at a point like this.

While power and torque are easy to calculate or extract from motor curves, efficiency is particularly difficult to accurately quantify. At low speeds and high torques, where losses are primarily due to I^2R resistive dissipation, the simple motor model can predict efficiency accurately. At the operating points in Figure 13a and Figure 13b, for example, the ratio of the power output rectangle area to the total rectangle area will give a good estimate of motor efficiency. At maximum power (Figure 13b), the efficiency will be nearly 50%, a general conclusion for any DC motor driven by a voltage source.

Where the efficiency estimate of the simple model breaks down is at the low-torque, high-speed operating points (Figure 13c). Here, other internal losses may become significant, such as friction and eddy current losses within the motor. These other internal losses will create some minimum torque demand even when the shaft is not externally loaded. The power going toward this torque is lost from the total output of the motor. Visually, this could be represented as an additional rectangular area of losses, as in Figure 14.

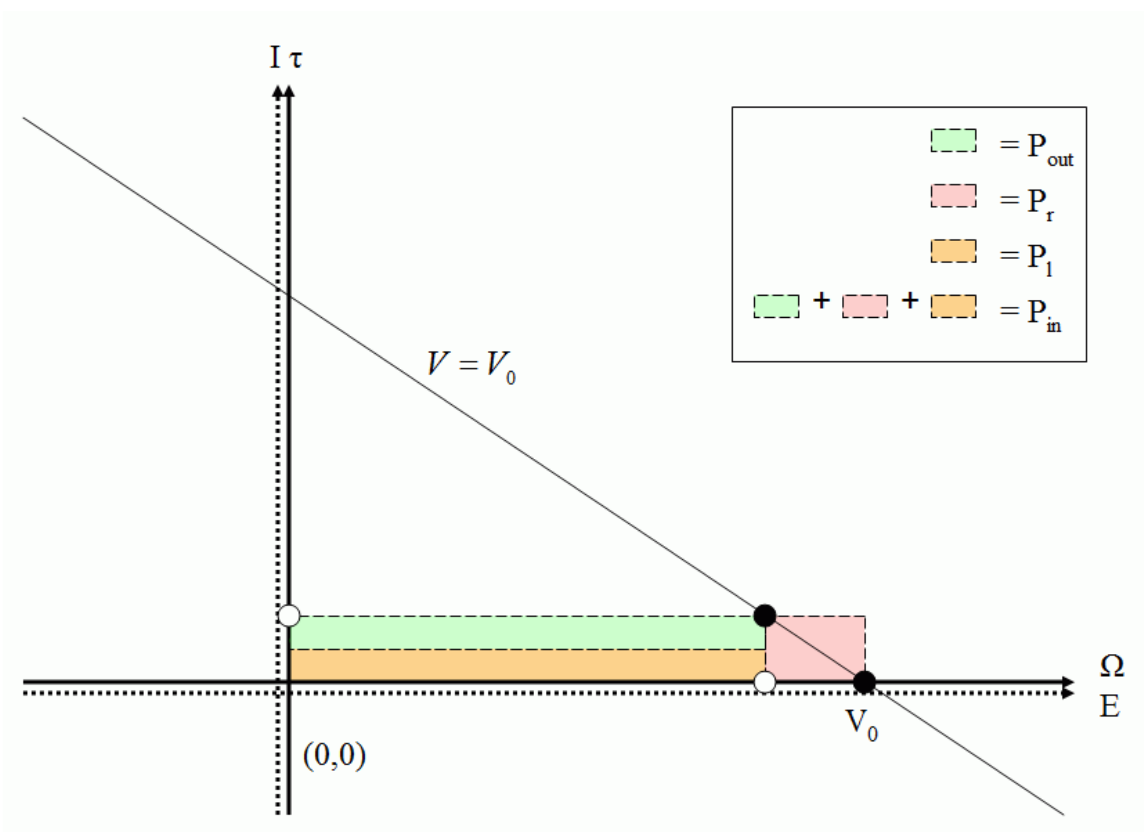

Figure 14: An operating point at high speed, re-drawn with other sources of loss accounted for.

The internal loads other than resistance are represented by the P_l rectangle. Even though they are small compared to the maximum power output of the motor, they can begin to dominate at high speeds. Peak efficiency will occur at some optimum point along the torque-speed curve between maximum power and no-load speed. At higher speeds, the internal loads begin to take back more than the reduction in I^2R loss gives. This gives the efficiency-speed curve a characteristic shape such as in Figure 15, where the internal losses are modeled as a constant torque.

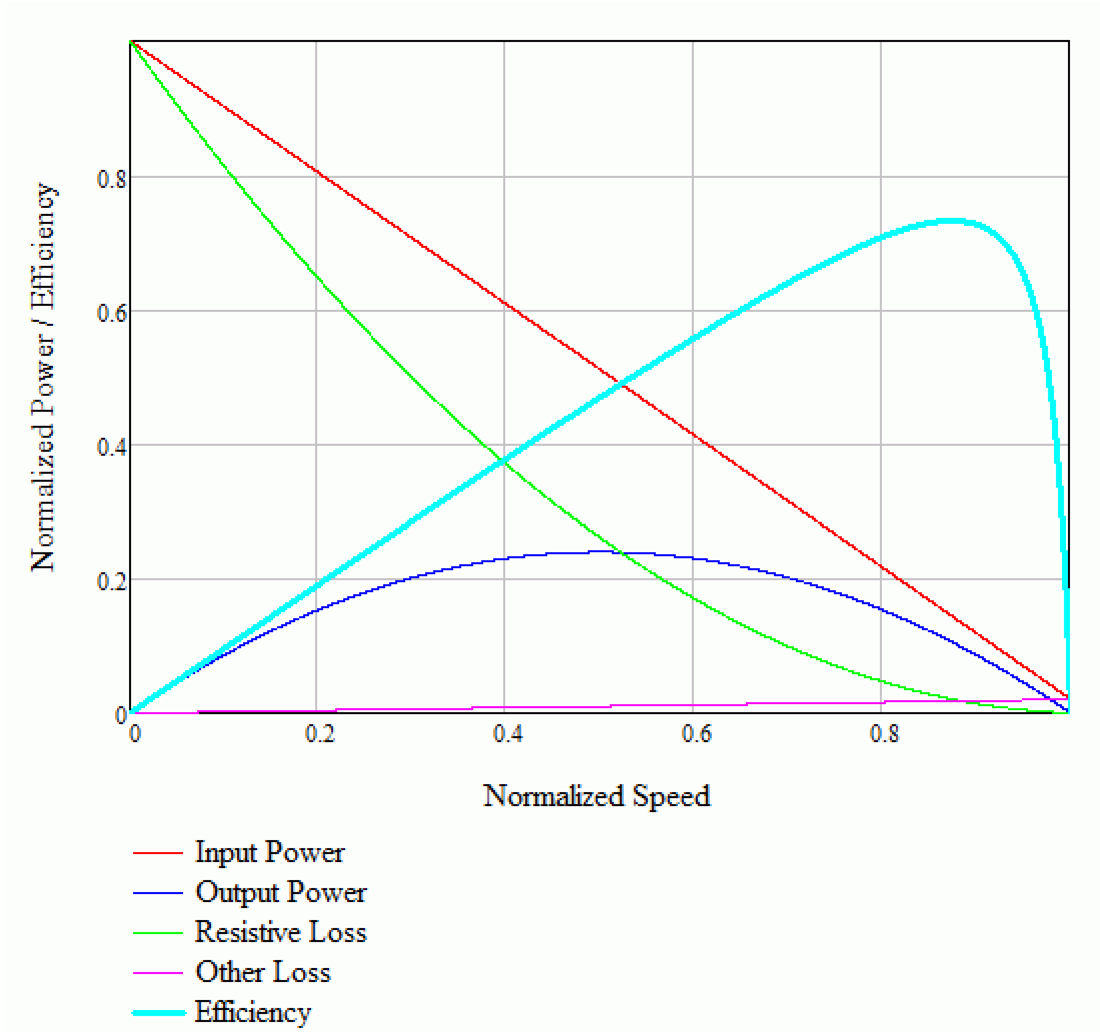

Figure 15: Normalized powers and efficiency in the presence of other losses, here modeled as a constant torque (Coulomb friction). This gives rise to the characteristic shape of the efficiency curve.

Constant torque is only one possible model of the other internal loads. It may be a good approximation if the other loads are dominated by Coulomb friction, for example. Other possible internal loads are listed in Table 3. A real motor will have some contribution from all of these loads. As a system designer, it is useful to know the sources of dissipation in the motor and the trends they follow, but establishing an all-inclusive model is not usually necessary. Motor manufacturers do typically provide an efficiency curve for reference.

Table 3: Some possible dissipative loads in the motor other than resistance.

Load	Description	Approximate Model
Coulomb Friction	Kinetic dry friction between the motor shaft and bushings, for example.	$\tau = \tau_0$
Viscous Damping	Might approximate a lubricated bearing inside the motor.	$\tau \propto \Omega$
Eddy Current Damping	Eddy currents in the windings and steel core of the motor create drag.	$\tau \propto \Omega$
Air Drag (Windage)	Shear drag in the air gap of the motor.	$\tau \propto \Omega^2$

Scaling Torque-Speed Curves

The torque-speed curves used in the previous analysis were all based on a fixed voltage applied across the motor terminals. Motor manufacturers will typically give a nominal voltage at which the torque speed curves published for a given motor are measured. However, it is useful to be able to scale the torque-speed curve for different voltages. The battery or supply used in a particular application may not exactly match the voltage of the given torque-speed curve, or the motor may be driven by a variable-voltage output electronic speed controller.

Since the torque-speed curve is linear, only two points are needed to define a new relationship at a higher or lower operating voltage. The most convenient points to use for scaling are the stall torque and the no-load speed, given in equations (22) and (26), respectively. Both are proportional to the operating voltage, V_0. Thus, doubling the operating voltage would double the stall torque *and* the no-load speed. The maximum power would increase by a factor of four. Figure 16 shows this graphically.

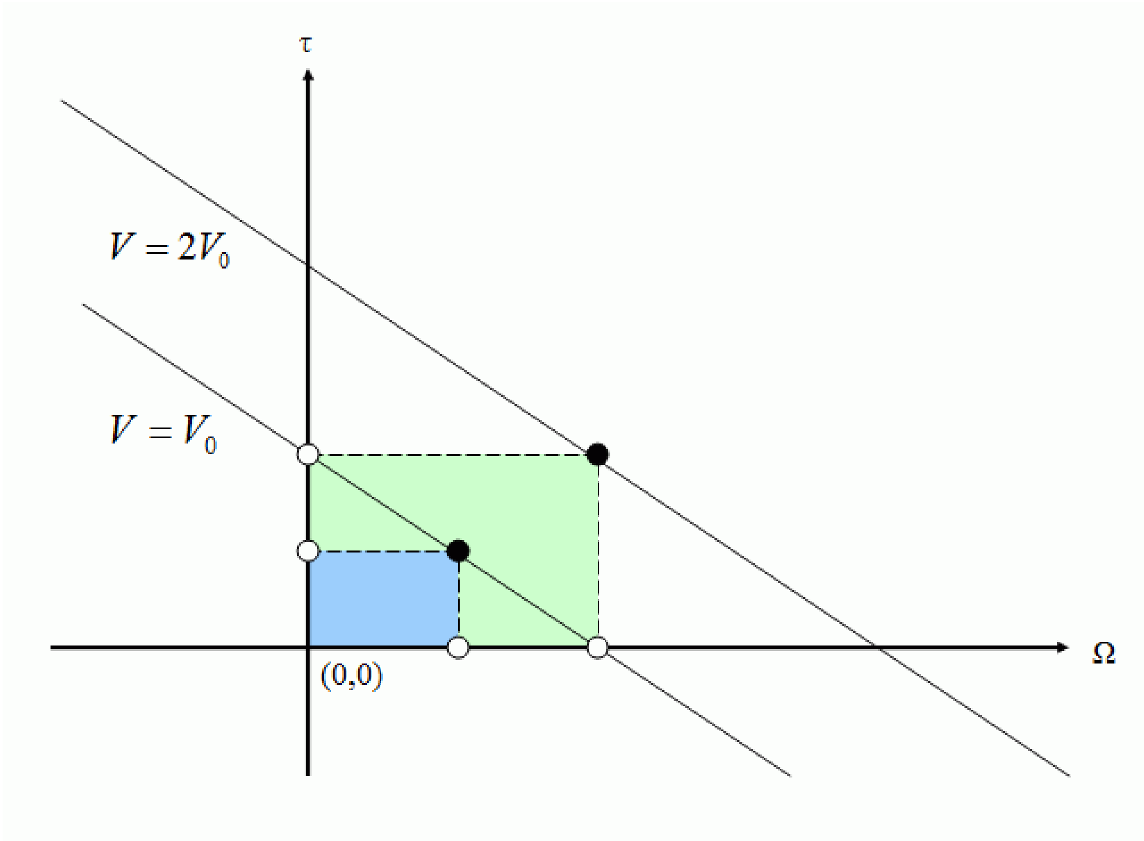

Figure 16: Both the stall torque and the no-load speed double if the operating voltage is doubled. The maximum power, represented by the areas of the rectangles, quadruples.

3.2.3 Failure Mechanisms

Mechanical Failure Mechanisms

As a mechanical element, motors are subject to the same guidelines for acceptable loading and stressing as any other component in rotary motion applications. Almost invariably, the motor shaft will be designed to sustain the maximum rated torque plus a considerable safety factor. However, it is probably *not* designed with any consideration for bending moment. The safest course of action, then, is to couple a motor to your system in a way that allows the shaft to transmit torque only.

The most obvious violation of the "transmit torque only" rule is the case where a cantilevered load is directly coupled to the motor shaft, such as in Figure 17a. Here, the normal force on the wheel creates a bending moment on the motor shaft which will increase frictional losses at the motor bushing or, in the worst case, damage the motor shaft. A better configuration, shown in Figure 17b, is to use bearings and a separate shaft to counteract the normal force of the wheel.

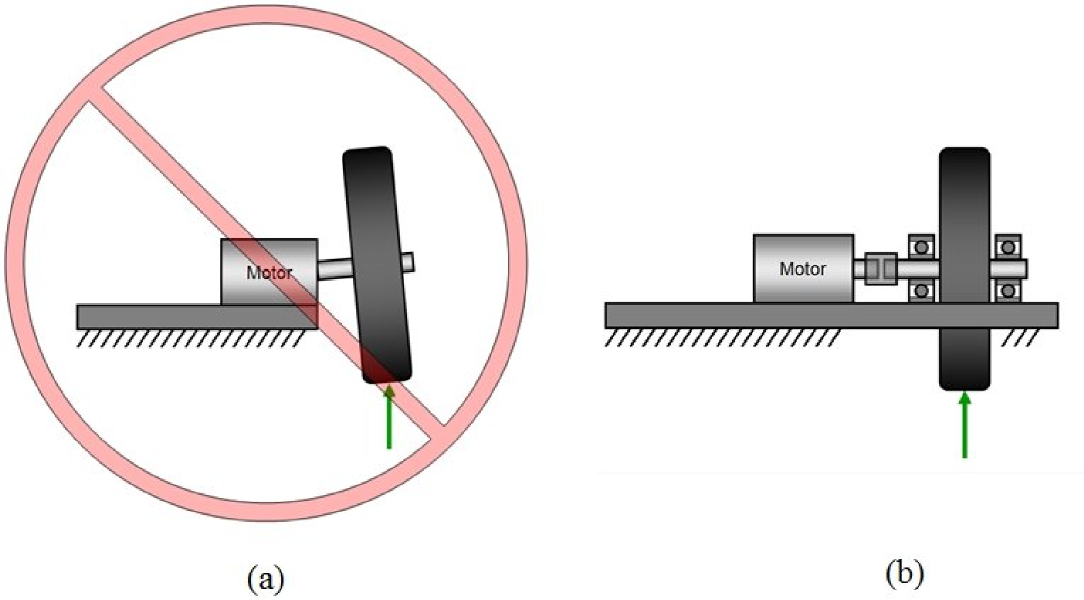

Figure 17: Motor shafts are generally not designed to take bending moments. A less risky configuration is to use separate bearings to support the load and allow the motor shaft to transmit only torque.

Using a separate, fully-constrained shaft to support the load creates another problem, though: misalignment between the motor shaft and the load shaft may also cause bending or shear stresses in the motor shaft. Depending on the rigidity of the structural loop from the load shaft to the motor shaft, these moments may just decrease motor performance through added friction or they may actually damage the shaft. Where extremely precise alignment of shafts is not possible, flexible couplings designed to transmit torque through some angular or parallel misalignment can help.

Electrical Failure Mechanisms

The most common electrical failure mechanism for a motor is overheating of the windings. Specifically, the insulation melts off the individual turns of wire in the winding, causing them to short-circuit. Wire insulation is classified by the maximum temperature rating during operation, from Class A (105ºC) to Class H (180ºC).

The steady-state temperature of the winding depends on the heat generated (I^2R losses), as well as the heat removed through conduction, convection, and radiation. When the two are in balance, the motor will run at a steady temperature indefinitely. Since this is a complex system with fluidic, electrical, and mechanical components, the steady-state performance of an electric motor is actually difficult to predict. It is more readily determined experimentally. Generally speaking, though, steady-state performance is limited by heat removal. Better heat sinking or convective cooling can most directly improve steady-state power capability of a given motor.

The transient thermal performance of an electric motor is somewhat easier to predict. Assuming that the motor can transfer heat within itself much faster that it can transfer heat to the outside environment, it can be modeled as a lumped thermal capacitance. The specific heat capacity to use would be based on the weighted average of materials in the motor. Since the exact ratio of materials is generally not known, a good starting point might be an average of the specific heat capacity of copper and steel, which make up the bulk of the motor's mass.

Table 4: Specific heat capacities of copper and steel, the largest fraction of a motor's mass.

Material	Specific Heat Capacity
Copper	$C = 0.39\frac{kJ}{kg \cdot K}$
Steel	$C = 0.49\frac{kJ}{kg \cdot K}$
Average	$C = 0.44\frac{kJ}{kg \cdot K}$

Using the lumped thermal capacitance, the calculation of transient current capability is straightforward. A motor with a resistance, R, mass, m, and starting temperature, T_0, could operate with a given current, I, for a maximum amount of time, t_{max}, before the energy dissipated in the resistance increases the temperature of the motor to a maximum allowable temperature, T_{max}. The following equation can be used to calculate t_{max}, given the other quantities:

$$t_{\max} = \frac{mC(T_{\max} - T_0)}{I^2 R}. \tag{28}$$

3.3 Manufacturing

3.4 Selection

In most cases, an electric motor is just one part in a large system with many variables. Thus, it is impossible to consider the selection of a motor for an application without considering the constraints of the entire system. Often, the system consists of a number of power transforming elements "in series," with more design variables than strictly needed.

For example, consider a system consisting of a motor, a single-stage gear reduction, and a wheel. The motor converts electrical power into rotary mechanical power. The gear reduction converts from one torque and speed to another. The wheel converts from rotary mechanical power to linear mechanical power. If all three elements are ideal and lossless, there are three variables to choose in the design: the torque constant of the motor, the gear ratio, and the radius of the wheel. Figure 18 shows this system and the power transformations involved.

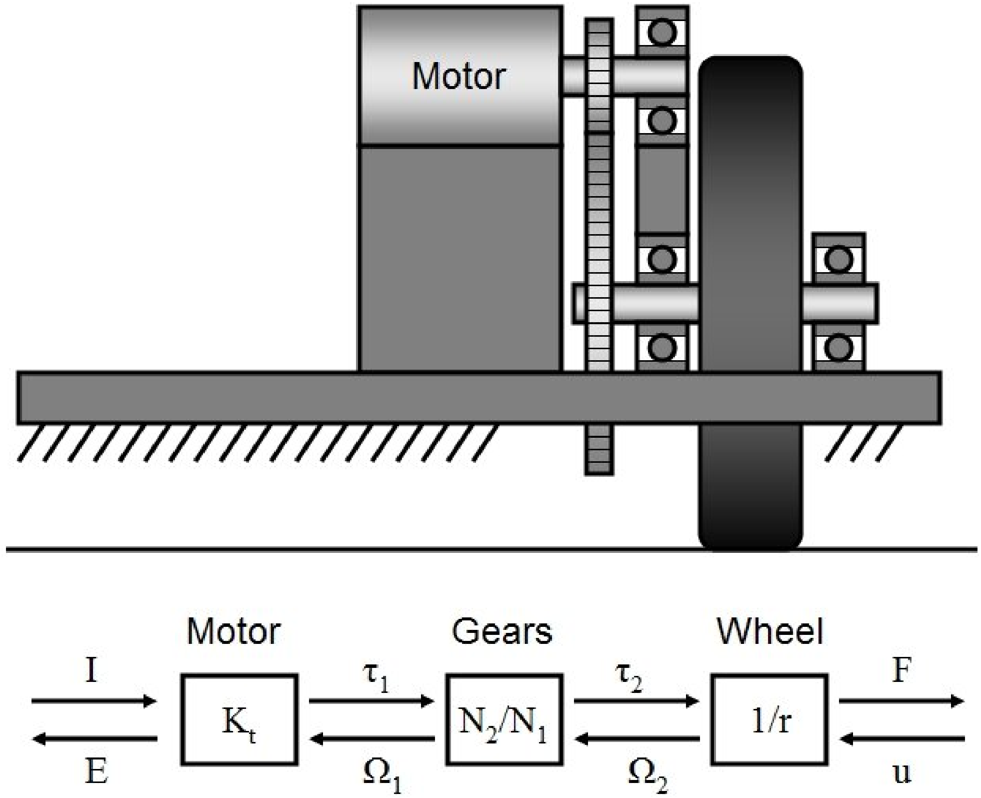

Figure 18: A system comprising a motor, single-stage gear reduction, and a wheel transmitting force to the ground. The overall transmission ratio from current to force and from velocity to back EMF is the sum of three components.

However, the *overall* transformation from current and voltage to force and velocity is the product of all three variables. Thus, a change in one may be substituted for a change in the other with the same net effect. A large wheel does the same job as a smaller gear reduction, or a motor with a lower torque constant. The choice of which variable to change must be made based on other considerations. What motors are available? What gear ratios? How big or small can the wheel be? All the components in the system are considered together.

Even if the other system parameters (gear ratio, wheel size, etc.) are relatively free, the motor selected must meet a few criteria. First, it must be able to produce sufficient power, accounting for all losses in the system, to accomplish the desired task. Even if the desired task requires a single, short burst of power, choosing a motor with a maximum power several times larger than the power required to perform the final task is usually best. It provides extra power to overcome losses downstream, such as in gears, bearings, linkages, or other components. Additionally, it allows the motor to operate more efficiently, at reduced torque and power.

By choosing a motor and setting the other system parameters (gear ratio, etc.), an operating point on the motor's torque-speed curve is defined. The final load, reflected through the transformation ratios of each system component, dictates a torque and speed requirement of the motor itself, which must lie on the torque-speed curve of the selected motor at a voltage achievable by the power supply. Generally, placement of the operating point somewhere between maximum power and maximum efficiency is best. Figure 19 shows a guideline for placing the operating point of the motor.

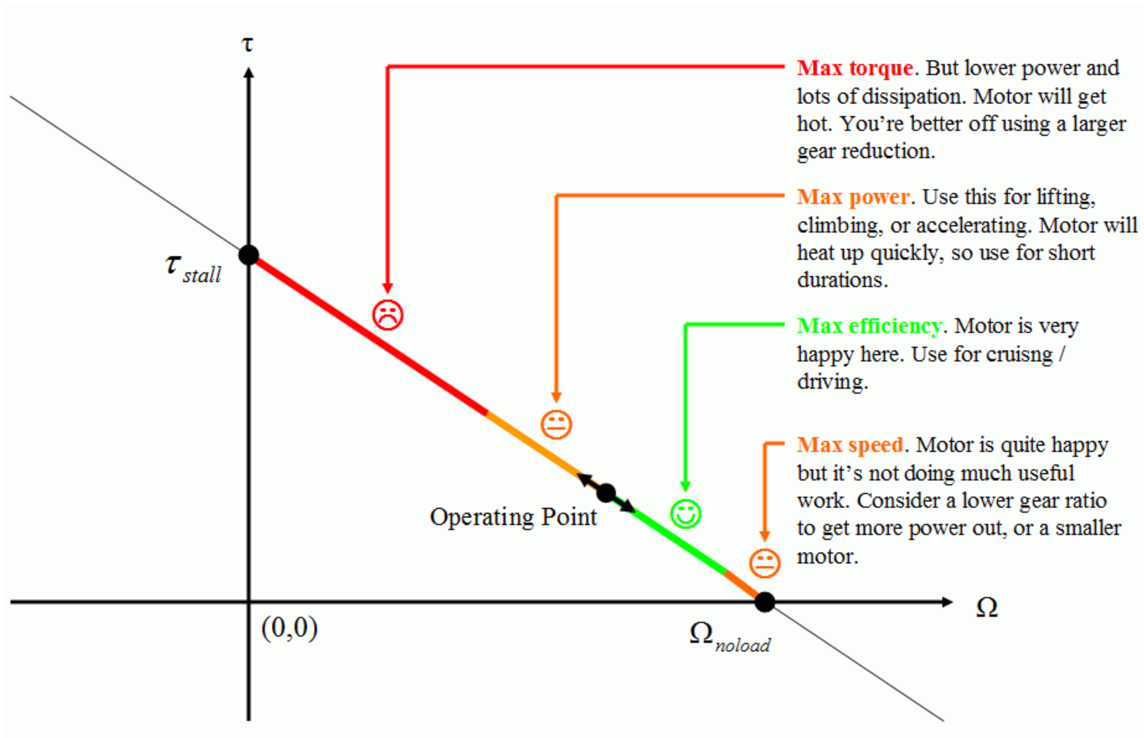

Figure 19: Placing the motor's operating point somewhere between maximum power and maximum efficiency is usually best.

3.5 Hand-On Exercise

3.6 Problems

Problems 1-7 refer to the motor represented by the ideal torque-speed curve in Figure 20, which is given at 12V.

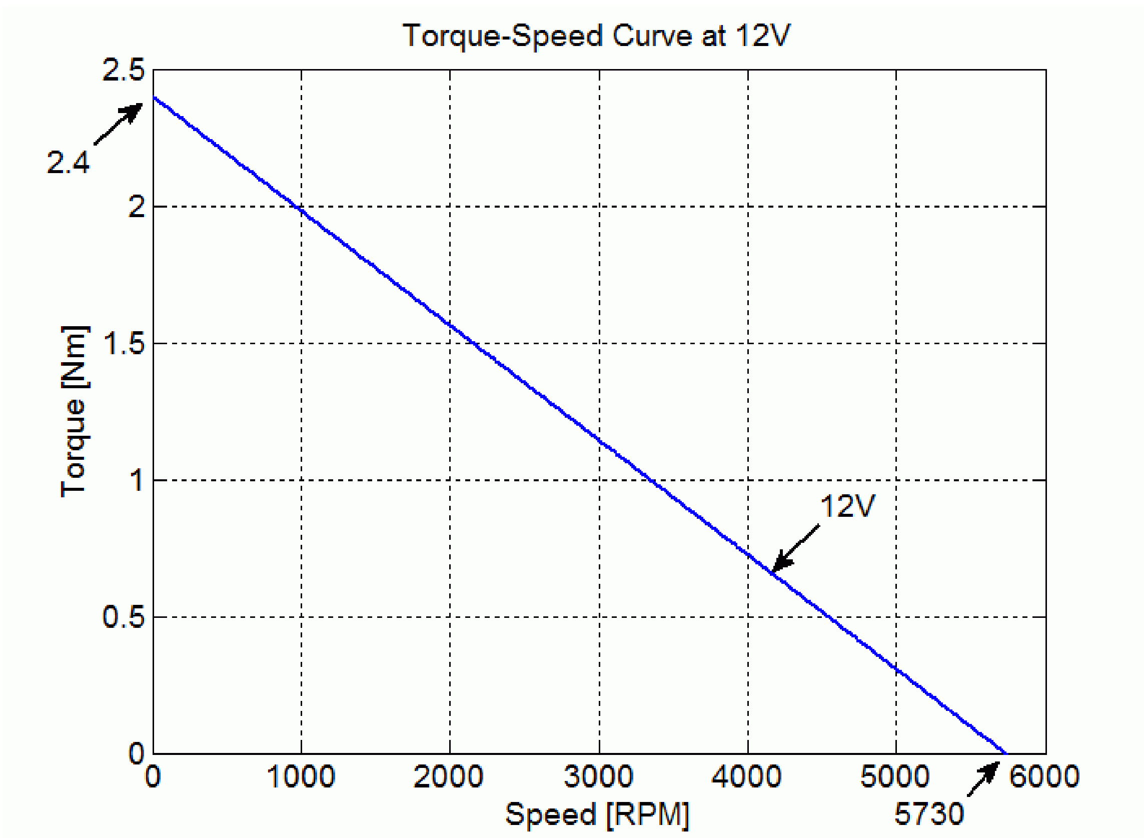

Figure 20: Torque-speed curve for reference in Problems 1-6.

1. Operating at 12V, what are the torque constant and resistance of the motor?

Answer:
The torque constant can be calculated from the no-load speed and voltage, since it is also the constant of proportionality between back EMF and speed. (At no load, back EMF and applied voltage are equal.) From Equation (26):

$$K_t = \left(\frac{V}{\Omega_{noload}}\right) = \left(\frac{12V}{5730rpm}\right) = \left(\frac{12V}{600\, {rad}/{s}}\right) = \left(0.020\frac{V}{{rad}/{s}}\right) = \left(0.020\frac{Nm}{A}\right).$$

With the torque constant known, the stall current can be found. The resistance is just the applied voltage divided by stall current (no back EMF at stall). This is summarized in Equation (22).

$$R = \frac{K_t V}{\tau_{stall}} = \frac{\left(0.020\frac{Nm}{A}\right)(12V)}{2.4Nm} = 0.10\Omega\,.$$

2. Operating at 12V, what is the maximum power output of the motor?

Answer:
Maximum output power occurs at half the no-load speed and half the stall torque. Using the given values:

$$P_{\max} = \left(\frac{\Omega_{noload}}{2}\right)\left(\frac{\tau_{stall}}{2}\right) = \left(\frac{600\,{}^{rad}\!/_{s}}{2}\right)\left(\frac{2.4Nm}{2}\right) = 360W\,.$$

Another way to calculate the maximum power is with voltage and resistance, as in Equation (27):

$$P_{\max} = \frac{V^2}{4R} = \frac{(12V)^2}{(4)(0.10\Omega)} = 360W.$$

3. Operating at 6V, what are the torque constant and resistance of the motor?

Answer:
The torque constant and resistance do not vary with operating voltage. They are the same as in Problem 1.

$$K_t = 0.020\frac{Nm}{A}\,.$$
$$R = 0.10\Omega\,.$$

4. Operating at 6V, what are the stall torque, no-load speed, and maximum power output of the motor?

Answer:
Both stall torque and no-load speed are proportional to applied voltage. If the applied voltage is halved (from 12V to 6V), both the stall torque and the no-load speed will be halved:

$$\tau_{stall\_6v} = \frac{\tau_{stall\_12V}}{2} = 1.2Nm\,.$$
$$\Omega_{noload\_6v} = \frac{\Omega_{noload\_12V}}{2} = 2865rpm = 300\frac{rad}{s}.$$

Maximum power output is proportional to the product of stall torque and no-load speed. If each of these quantities if halved, the maximum power will be divided by four:

$$P_{\max\_6V} = \frac{P_{\max\_12V}}{4} = 90W.$$

5. The motor, operating at 12V, will be used to lift a 2kg mass by directly attaching it to a winch pulley (see Figure 21). What pulley radius will result in the fastest *steady-state* lift rate? What is the lift rate with this pulley fitted? (Assume no losses in the pulley mechanism itself.)

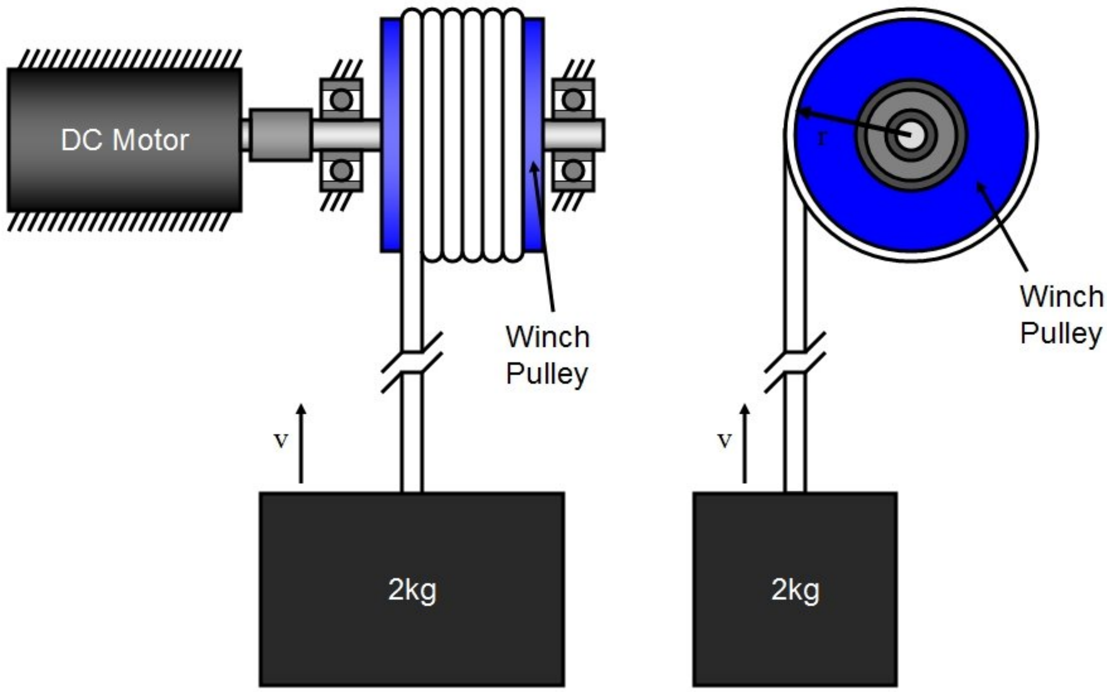

Figure 21: The DC motor used to drive a winch pulley lifting a 2kg mass.

Answer:
There are a few ways to solve this problem. One is to consider the second question first: What is the maximum steady-state lift rate? This can be solved independently just by knowing the maximum power output of the motor operated at 12V, which was calculated in Problem 2 to be 360W. Assuming no losses in the winch mechanism itself, all the output power goes into lifting the 10kg mass. The velocity is:

$$v = \left(\frac{P_{\max}}{F}\right) = \left(\frac{P_{\max}}{mg}\right) = \left(\frac{360W}{2kg \cdot 9.8 \, m/s^2}\right) = 18.4 \, m/s \, .$$

The motor puts out maximum power at half the no-load speed. Knowing this, the pulley radius that corresponds to the maximum lift rate calculated above can be found:

$$r = \frac{v}{\left(\frac{\Omega_{noload}}{2}\right)} = \left(\frac{18.4 \, m/s}{300 \, rad/s}\right) = 0.061m \, .$$

Another way to solve the problem is to find the radius of the winch pulley that will exactly stall the motor, then divide by two. That approach will also give the maximum power operating point.

$$r_{stall} = \frac{\tau_{stall}}{mg} = \left(\frac{2.4Nm}{2kg \cdot 9.8 \, m/s^2} \right) = 0.122m \, .$$

$$r = \frac{r_{stall}}{2} = 0.061m \, .$$

Using this, and half the no-load speed, the maximum lift rate can be found.

6. The motor weighs 1.3kg, with an average specific heat capacity of 0.44kJ/kg-K, and has Class H insulation (180ºC maximum operating temperature). If the motor starts at 25ºC and is operated at 12V, estimate how long it can operate at stall. Estimate how long it can operate at maximum power. (You may assume no heat transfer out of the motor as a worst-case scenario.)

Answer:
Assuming no heat is transferred out of the motor, the time to reach maximum operating temperature is a function of the power dissipated in the motor windings (I^2R) and the total heat capacity of the motor. This is summarized in Equation (28). First, the calculation can be done using the stall current:

$$t_{stall} = \frac{mC(T_{\max} - T_0)}{I_{stall}^2 R} = \left(\frac{1.3kg \cdot 0.44 \frac{kJ}{kg \cdot K} \cdot (180 - 25)K}{\left(\frac{12V}{0.1\Omega} \right)^2 0.1\Omega} \right) = 62s \, .$$

This assumes uniform temperature distribution in the motor, which is not likely to be the case. The windings, where heat is being generated, will be hotter than the surrounding steel and motor case, and may exceed 180ºC well before 62 seconds.

At maximum power, the current is halved and the dissipated power is lowered by a factor of four. Since the heat capacity doesn't change, the motor can operate at maximum power for four times longer, or about **4 minutes**.

7. Running the motor at maximum power may be reasonable if it only needs to perform lifts intermittently. What change could be made if it needs to run continuously 24 hours a day? What tradeoff is there?

Answer:
Running continuously at maximum power may damage the motor. If there is no heat transfer out of the motor, it could only run at maximum power for four minutes, as

calculated in Problem 6. However, it might be possible to create enough heat transfer using active cooling (fans or liquid) to pull at least 360W of heat out of the motor continuously.

Even if continuous cooling of 360W could be provided, the motor would be running at less than 50% efficiency. A better solution would be to shift the operating point of the motor to a more efficient location by decreasing the torque and increasing the speed of the motor. This can be done by **making the pulley radius smaller**. The motor will no longer be running at maximum power (so **the lift rate will decrease**), but it will be able to run continuously without overheating.